

**CONTROL VALVES
IN TURBO-COMPRESSOR ANTI-SURGE SYSTEMS**

kentintrol

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KOSO

KOSO KENT INTROL SUPPLIES A DIVERSE RANGE OF PRECISION -MANUFACTURED CONTROL, CHOKE, AND ROTARY VALVES FOR OIL AND GAS, PETROCHEMICAL AND POWER INDUSTRIES – WORLDWIDE



PRODUCT RANGE

TOP & BOTTOM-GUIDED CONTROL VALVES

Our range of high-performance top and bottom-guided control valves includes single and double-seated valves suitable for low and high-capacity applications, as well as three-way valves for mixing or splitting flows. Our control valves are designed to facilitate pressure drops at all stages of transportation in the oil, gas and power industries. All valves are refined by our engineers to suit the needs of each application and all service conditions.

CAGE-GUIDED CONTROL VALVES

The Series 1200/7200 range of cage-guided control valves is KKI's core product. The exceptional valves in this range combine high-integrity features, such as ASME VIII body/bonnet bolting design, a high flow capacity and a wide range of trim designs, from low-noise anti-cavitation to multi-stage trims. These valves are ideally suited to the critical service process control requirements of a wide range of industry applications.

SURFACE CHOKE VALVES

The KKI Series 73 surface choke valve offers a unique solution for the majority of choke applications in the oil and gas industry. The flexible valve design can incorporate many different trim and body material options to suit differing flow rates and in-service conditions. Thousands of KKI Series 73 surface chokes are installed around the world on projects for some of the world's leading oil and gas production companies.

ROTARY CONTROL VALVES

The Rotrol range of high-capacity butterfly valves has been developed to overcome the problems associated with control, cavitation and noise in conventional butterfly valve designs. Lighter in weight and more compact than globe valve alternatives, this innovative valve performs especially well in severe-service applications, where pressure drops tend to be high in the controlling position but where high-capacity throughputs at low pressure drops are also required.

SEVERE SERVICE SOLUTIONS

For more than 50 years, KKI has built up a reputation for delivering valve solutions for the most arduous service conditions. We have developed a range of advanced, high-quality severe service valve solutions for every type of problematic application, from high-pressure, high-temperature environments to sub-zero temperatures. Our valves are designed to combat the effects of cavitation, flashing, erosion, contaminated fluids, corrosion, high velocity, vibration, noise and energy dissipation.

INSTRUMENTS

KKI offers a wide selection of sophisticated instrumentation to support our comprehensive range of high-performance valves and actuators. The instruments we supply include pneumatic and electro-magnetic positioners, airsets, volume boosters and airlocks. All instruments are specified to deliver optimum performance for the service conditions and specific needs of each application. We also supply proprietary instruments to suit individual customer preferences.

ACTUATORS

Our range of robust, versatile and reliable pneumatic actuators includes the 'G', 'C' and 'D' Series models. These have been developed to meet the needs of all control valve applications, offering proven design and high reliability. They are used extensively for on-shore, offshore and power installations. In addition, we supply various proprietary actuators – such as electric, electro-hydraulic, pneumatic stepping and hydraulic stepping actuators – to meet customer requirements. All actuators can be supplied with hand-wheels and limit stop features.

TOP & BOTTOM-GUIDED CONTROL VALVES

SINGLE SEATED SERIES 10/71



DOUBLE SEATED SERIES 20



3-WAY MIXING AND DIVERTING SERIES 30



CAGE-GUIDED CONTROL VALVES

SERIES 1200/7200



SURFACE CHOKE VALVES

SERIES 73

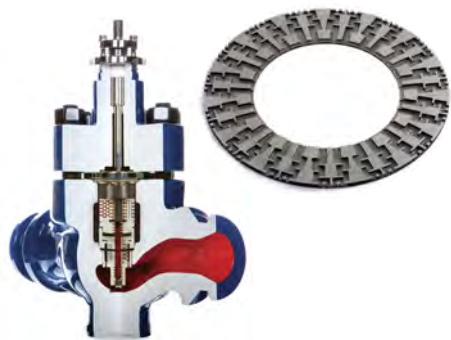


ROTARY CONTROL VALVES

SERIES 60



SEVERE SERVICE SOLUTIONS



INSTRUMENTS



ACTUATORS

SERIES G



SERIES C



SERIES D



FOREWORD



When considering the application of control valves to turbo-compressor anti-surge systems, the service requirements are usually beyond the capabilities of the standard valves. Off-the-shelf valves are most unlikely to satisfy the excessive demands of these applications.

In these cases, the required severe service valves need to be individually designed, and collaboration between the plant designers, the valve engineer and the operating engineers is necessary.

This paper addresses the unique problems that exist in the application of control valves for compressor anti-surge systems. It identifies the nature of these problems, and provides assistance in the specification of the control valve that will be required to overcome the specific service demands.

An introductory guide, it is aimed at engineers with little or no previous experience in this aspect of control engineering. Further assistance is available via Koso Kent Introl, experts in innovative design technology and skilled engineering in the severe service valves market for over 45 years.

ED SINGLETON

THE AUTHOR

Edward ("Ed") Singleton began his working life as an apprentice in West Yorkshire, England in 1941. A Chartered Engineer, he graduated with an Engineering Degree from London University in 1950. A few years later he gained the professional qualification of Fellow of the Institution of Mechanical Engineers. His experience with control valves extends over the past six decades. He has been a true pioneer in the field, founding Introl Ltd in the late 1960s, which later joined forces with the Kent Group before being bought by Japan's Koso Group in 2005.

Ed retired as Managing Director in 1990 and has acted in the capacity of Consultant to Koso Kent Introl since that time. Ed also represented the UK on the IEC International Control Valve Standards Committee from 1990 until 2004. In 1995 he was awarded the prestigious Certificate of Merit by the BVAMA for contributions to control valve technology.

His unique expertise, built up from the vast array of world-wide control valve projects on which he has worked, is shared in a series of papers, of which this is the latest. Covering a wide variety of control valve topics including other tailored-specification valve uses such as power station boiler feedwater systems and high pressure pump recirculation services, all of Ed's papers are available from Koso Kent Introl on request.

February 2010



THE APPLICATION OF CONTROL VALVES TO COMPRESSOR ANTI-SURGE SYSTEMS

THE PROBLEM OF COMPRESSOR SURGE

Pipelines transporting gases and vapours are invariably dependent on centrifugal or turbo-compressors for the propulsion of these fluids. Under normal operation, with the compressor running at any constant speed there is a specific relationship between the pressure head across the compressor and the flow through it. But this stable relationship can be disturbed by sudden changes in flow, pressure and density, usually caused by sudden variations in demand downstream of the compressor or in the case of systems requiring multiple compressors, a disturbance can be caused by the switching of compressors in and out of service. All these can give rise to formidable pulsations of pressure and flow, better known as a surge. Under surge conditions the compressor may run erratically and a situation can arise where the pressure build-up in the downstream pipe may overcome the delivery pressure of the compressor, resulting in a flow reversal, reversing the compressor and causing mechanical damage.

A surge is usually preceded by small fluctuations, followed by a modest but very sharp drop in flow, to be followed by a massive drop at the commencement of the surge. The flow and pressure pulsations then, if unchecked, settle down to regular cyclic oscillations.

Surges may also develop at start-up and whenever the compressor is required to operate during periods of very low flow. Under these conditions, the compressor finds the flow too low for conversion to the required discharge pressure. The pressure in the discharge pipe then exceeds the impeller outlet pressure and this situation generates backflow which changes to forward flow when, in the next phase of the surge, the discharge pressure falls below the impeller outlet pressure. This cycle continues to produce unstable operation of the compressor along with excessive vibration. Steps must be taken to avoid this dangerous condition.

The usual method of preventing this condition is to include a control valve in the discharge line, along with appropriate instrumentation to recycle the gas to the compressor suction line (see figures 1A and 1B). This ensures there is always an adequate minimum flow through the compressor to maintain operation above the surge point. In some systems, a discharge vent valve is included. This maintains the flow above the surge point by venting gas from the discharge line in those instances when there is very little demand from the downstream process.

In practice, the load changes can happen very quickly so the anti-surge instrumentation and valves must have a fast response. Times to travel from the fully closed to the fully open position will vary with the valve's rated travel but, typically, a valve with a travel of 510 mm may be required to open in less than two seconds and one with a travel of 120 mm should open in 0.75 seconds. Obviously a valve with a short travel has an advantage over one with a long travel in achieving short opening times, and 0.2 seconds has been achieved by short travel valves. Just as important as the valve's fast opening is the capability of the valve and its associated instrumentation to react rapidly to small transient fluctuations in flow and pressure. This subject will be investigated in more detail in the section *Response to transients*.



FIGURE 1A. TYPICAL COMPRESSOR ANTI-SURGE BYPASS SYSTEM SHOWING ALTERNATIVE ANTI-SURGE VENTING SYSTEM

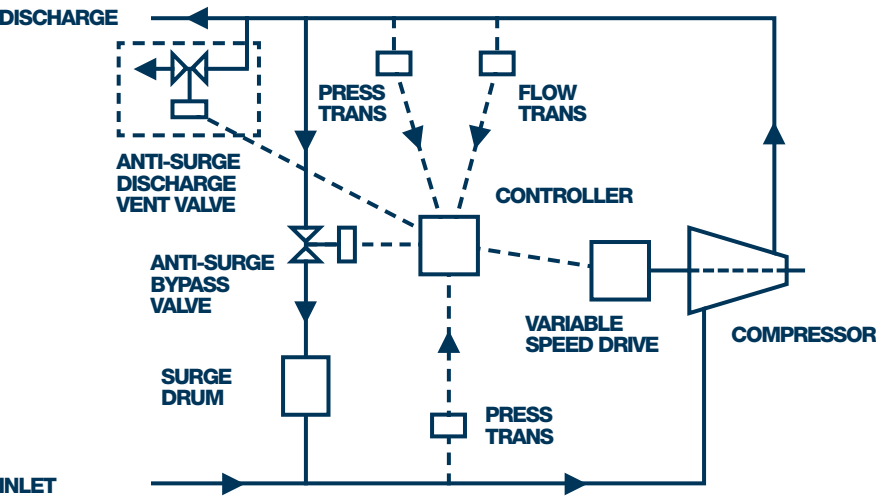


FIGURE 1B. 1. ANTI-SURGE CONTROL SYSTEM ISOLATED FROM PROCESS CONTROL

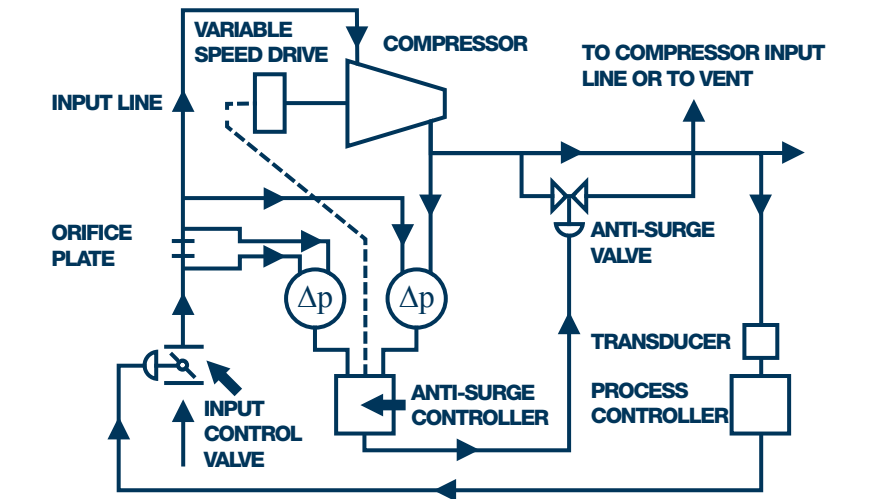
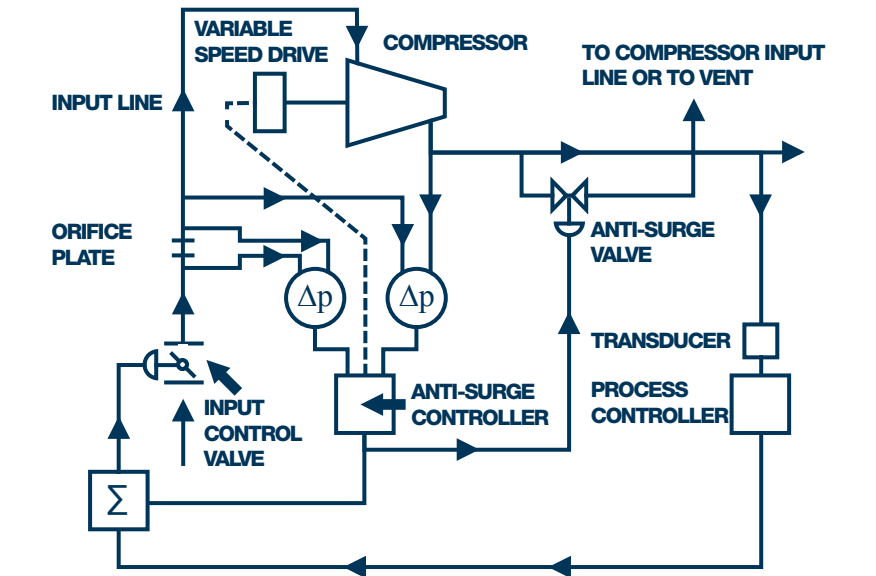


FIGURE 1B. 2. ANTI-SURGE CONTROL SYSTEM INTEGRATED WITH PROCESS CONTROL



ANTI-SURGE SYSTEMS

In Figures 1A and 1B, different arrangements of anti-surge instrumentation are shown, all having the same objective of instructing the valve to make the required adjustments to fluid flow and pressure, in order to prevent the onset of surge conditions. It is beyond the scope of this paper to discuss the merits of different control systems. However it cannot be overstressed that, whatever system is used, speed of response of the instrumentation is just as important as that of the valve.

EFFECTIVE SIZING

'Effective' rather than 'accurate' is used here intentionally to describe the 'art' of valve sizing for compressor anti-surge service. The control valves specified for most process control applications are sized for maximum flow conditions but, for anti-surge compressor service, the sizing must address the conditions that may be conducive to surges at different flow requirements and different compressor speeds.

Some of these factors are contradictory, in sizing terms. Valves on this service have to be able to handle surges in flow above the normal maximum flow required by the process. This may result in a larger valve than would be required for a normal process control application. Then, if the rated travel of the valve is not excessive, the higher flow/lift relationship of a larger valve might offer the added advantage of achieving significant flow changes in a very short time. Although the best methods and data available may be used, all these variables, some of which are not very precise, classify the sizing of compressor anti-surge valves as something of an art, rather than an exact science.

To understand the sizing problems, reference should be made to the flow versus pressure curves for the compressor, known as the compressor map (supplied by the manufacturer). Figure 2A is a simplified version of a map divested of everything except the surge line, the valve sizing line and the pressure delivery curves for various compressor speeds. If the operating point falls to the left of the surge line, the compressor will run into surge, and if the operating point falls to the right of the surge line the compressor will be operating under stable conditions. The anti-surge valve should be sized so that at all compressor speeds the operating point is to the right of the surge line. A working line is sometimes drawn on the compressor map indicating efficiency variations along each constant speed line. This is not shown in figures 2A and 2B but, more relevant to valve sizing, an arbitrary valve sizing line is shown. This is drawn to the right of the surge line, maintaining a gap of approximately 10 to 20% of the surge flow at all speeds. It therefore provides a safety margin against working conditions crossing the surge line.

If a control valve (linear characteristic) is sized on the conditions at point B (figure 2A), the resultant sizing curve will cross the surge line in the region of 75% to 80% of compressor speed, indicating unacceptable surge conditions at these speeds and below. If it is decided to use the compressor design point A as the datum for valve sizing, then the actual sizing curve will not cross the surge line until the speed drops to approximately 35 to 40%. This is an acceptable condition, but the valve is sized on a very high flow, making it quite large and therefore very expensive. The extra valve capacity is required at start-up conditions because of the low pressure drop available for the valve, but a more economical method of making this provision is preferable. Such a method is explained in the next paragraph when the complete compressor map (figure 2B) is considered.

FIGURE 2A. SIMPLIFIED COMPRESSOR MAP

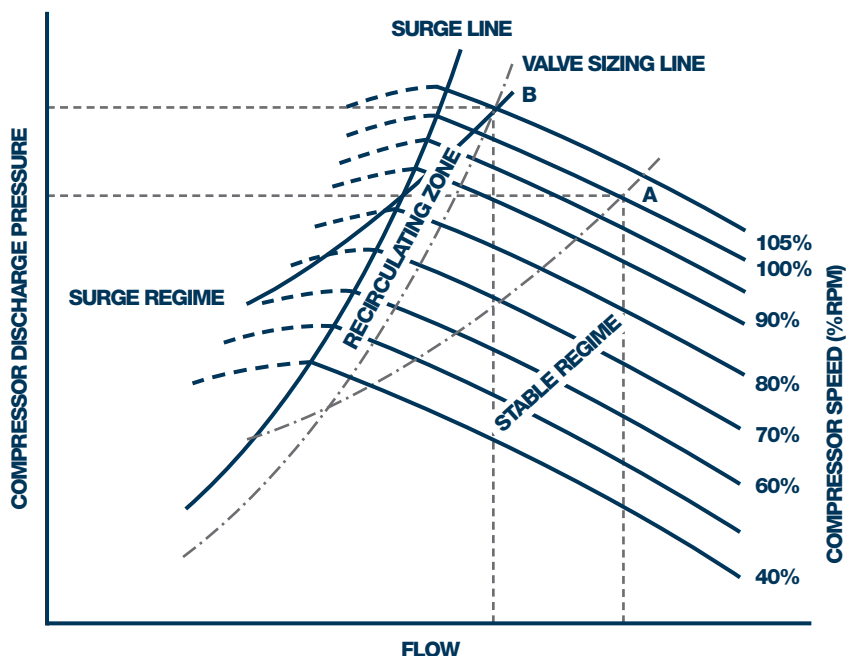


FIGURE 2B. COMPRESSOR PRESSURE/FLOW CHARACTERISTICS (COMPRESSOR MAP)

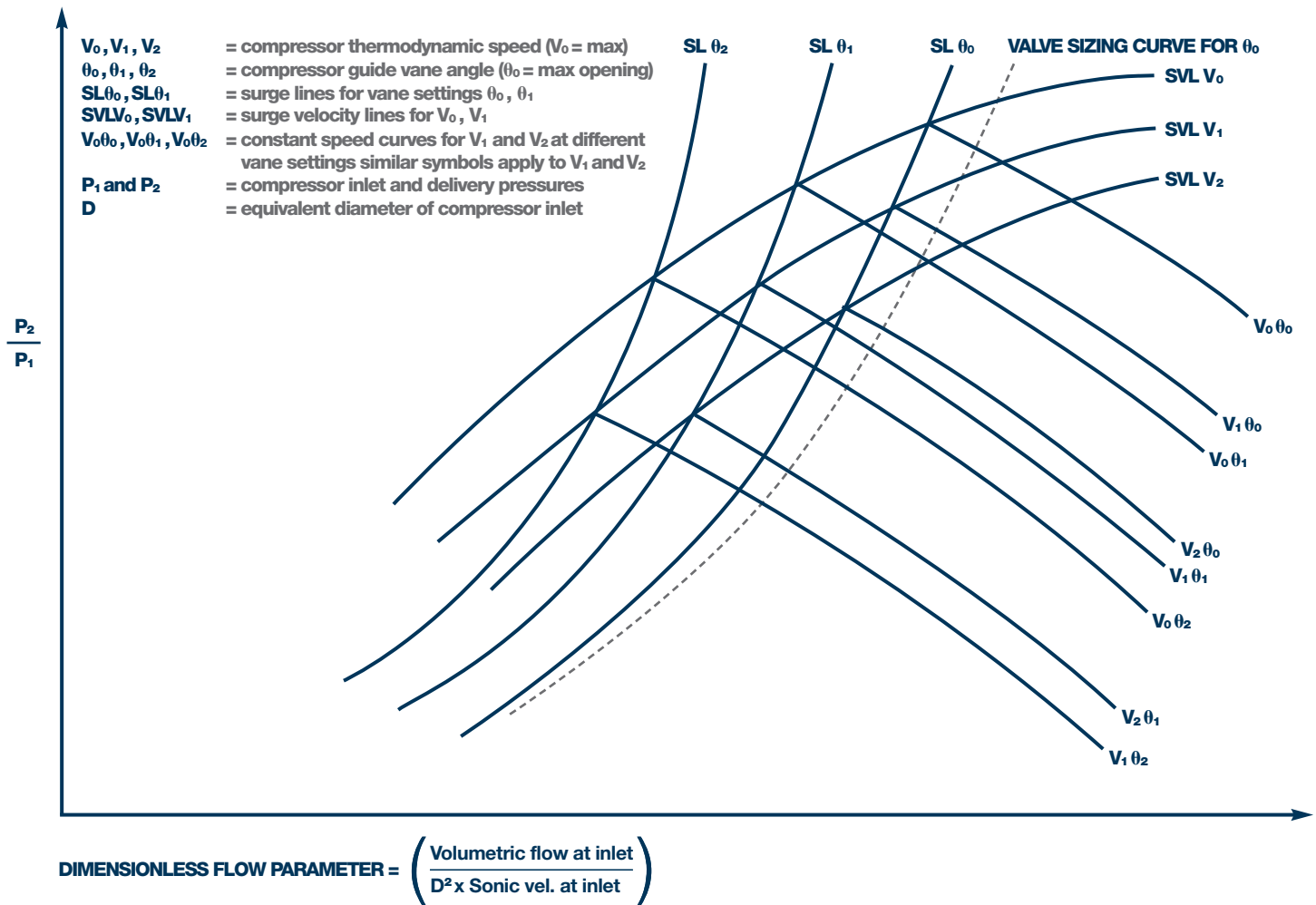


Figure 2B is an example of a complete compressor map including different thermodynamic speed curves and the effect of different angles of guide vanes when provided. The X axis represents flow and the Y axis the discharge pressure. For convenience, all terms are expressed in a dimensionless form:

Volumetric flow at inlet for the X axis, and $\frac{P_2}{P_1}$ for the Y axis

$$D^2 \sqrt{\gamma \frac{R}{M} T_1}$$

- D = equivalent diameter of the compressor inlet
 R = universal gas constant
 M = molecular weight
 T_1 = temperature at compressor inlet
 P_1 = pressure at compressor inlet
 P_2 = pressure at compressor outlet
 γ = ratio of specific heats C_p/C_v

The curved lines indicated by V_0, V_1, V_2 etc are different thermodynamic speed curves. Each is plotted for a constant value of:

$$V = \frac{n \times D}{\sqrt{\gamma \frac{R}{M} T_1}} \quad \text{a dimensionless term}$$

This equation indicates that the thermodynamic speed is influenced by the speed of rotation of the turbine (n) and the molecular weight and temperature of the fluid.

Considering each constant speed curve, and moving from a high to a low flow condition, it will be noted that P_2/P_1 increases until a point is reached, at the surge line, where the discharge pressure ratio P_2/P_1 starts to decrease. This is represented in figure 2A, but in reality the curves would start to be irregular after passing the surge line. At this point the flow becomes unstable and the compressor goes into surge. For different V curves (different values of one or all of n , T_1 , γ and M), there will be a different point on the compressor characteristic diagram where surge begins. Joining these points on each V curve gives the compressor surge line. The area to the left of this line is known as the 'surge regime' and to the right the 'stable regime'. If the compressor, through a sudden drop in demand from the process, should be constrained to operate in the surge regime it could, as already explained, sustain damage.

Some compressors are equipped with moveable inlet guide vanes for additional flow regulation (figure 2B). To simplify figure 2B, the constant speed curves have not been extended into the surge regime. Different guide vane angles, θ_0 , θ_1 , θ_2 , change the geometry of the compressor and this changes the V versus P_2/P_1 relationship. For each V curve there is a related q curve, i.e. $V_0\theta_0$, $V_1\theta_1$, $V_2\theta_2$ – and each will have a different surge point. Joining these surge points gives surge lines $SL\theta_0$, $SL\theta_1$, $SL\theta_2$ – for each guide vane angle. With increasing angles of the guide vanes, the surge lines move to the left of the chart, demonstrating that lower flows can be accepted before the surge mode is experienced.

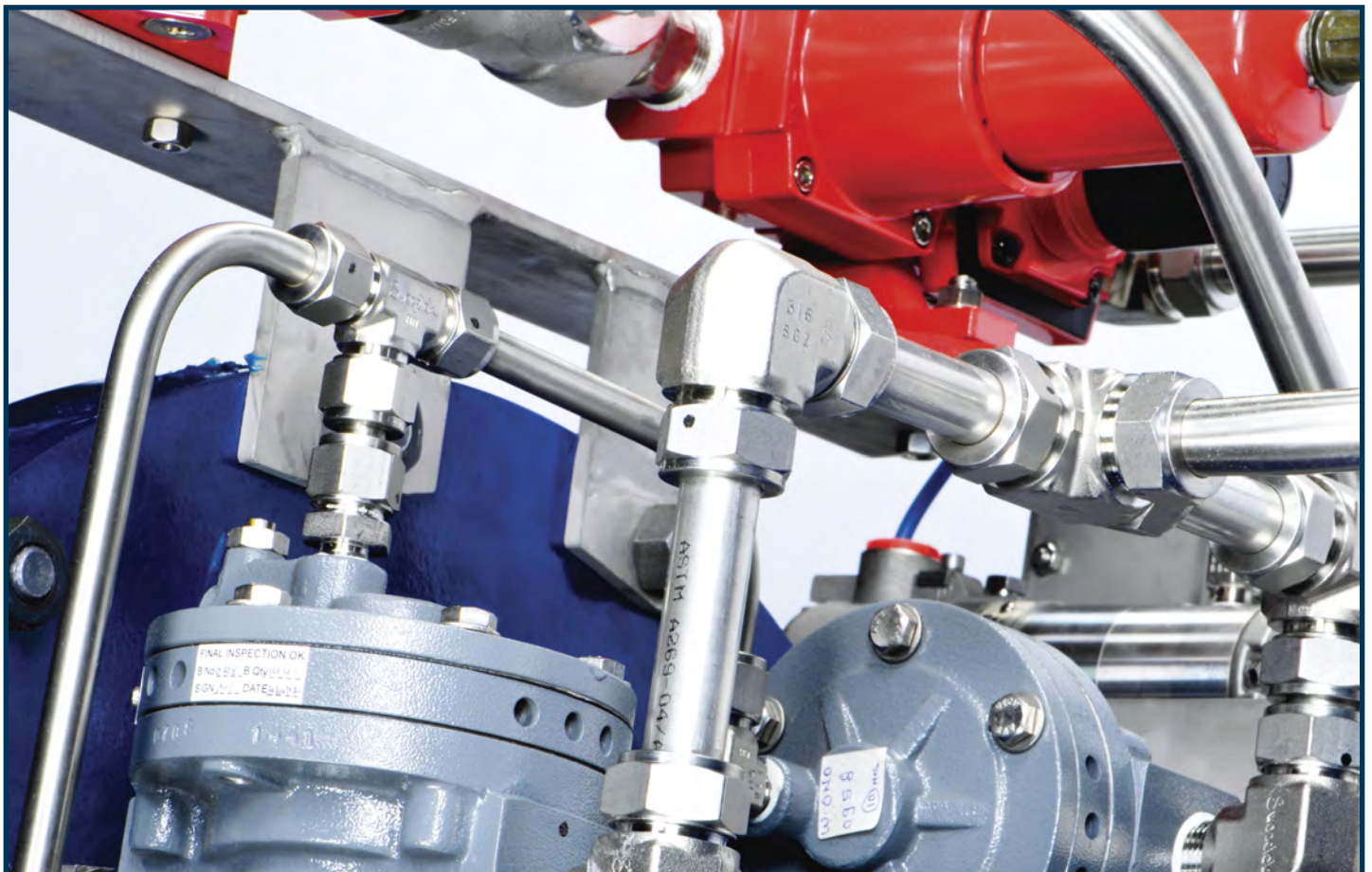
Changes in fluid temperature, molecular weight and to a lesser extent γ , produce changes in the thermodynamic speed V , although the compressor may be running at a constant speed.

An increase in temperature will produce a reduction in V (on the chart, perhaps from $V_0\theta_0$ to $V_1\theta_0$) and this gives a lower surge point. Similarly, an increase in molecular weight will produce an increase in V (on the chart, perhaps from $V_2\theta_0$ to $V_1\theta_0$) and this produces a higher surge point. Any such variations in working conditions must be taken into account when sizing the control valve.

To avoid surge conditions, the anti-surge valve must start to bypass flow just before any reductions in flow meet the surge line corresponding to the thermodynamic speed. The valve sizing line facilitates this requirement. It may be between 110% and 120% of the surge flow for each value of P_2/P_1 and for each value of θ , but the narrower the gap between the surge line and the valve sizing line, the smaller the valve and hence the lower the cost. Unfortunately all attempts to economise on valve size have to be abandoned when the most serious cases of surge are taken into account. Under these conditions the valve may be called upon to recycle or vent the maximum capacity flow of the compressor.

To make provision for these conditions, one accepted practice is to make an initial calculation of the valve size or the valve rated C_v (see the appendix for the definition of C_v).

To make this calculation the operating data, notably the valve pressure drop and the flow, existing at the point of intersection of the 100% speed curve with the surge line, is used. This calculated C_v is then multiplied by 2.2 to arrive at the required valve maximum C_v . The 2.2 is an empirical figure established through practical tests. Alternatively, the operating data at the point of intersection of the 100% speed curve with the valve sizing line may be used and the calculated C_v multiplied by 1.8.



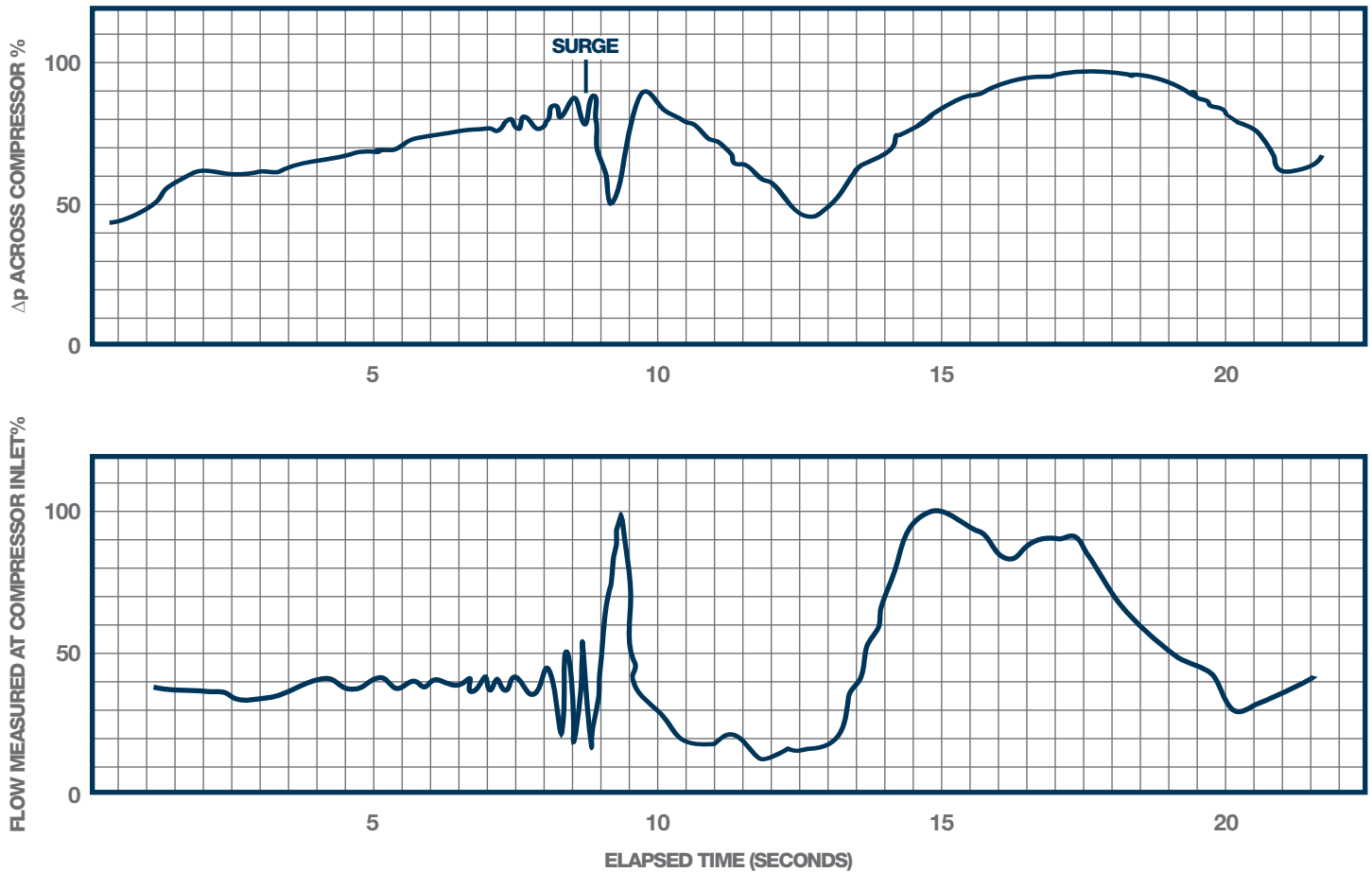
VALVE FLOW-LIFT CHARACTERISTIC

All control valves have a relationship between the flow and the lift or travel of the plug from the valve seat with the pressure drop across the valve remaining constant. The two most frequently used characteristics are linear and equal percentage. The linear characteristic is self explanatory. The equal percentage has an almost exponential relationship between flow and lift. Linear characteristic is more commonly used in turbo-compressor bypass systems because of its rapid increase in flow at low lift, but equal percentage is also used on occasions. On rare occasions, in order to ensure that the anti-surge valve operates effectively as near to the surge line as possible, special flow/lift characteristics are designed.

RANGEABILITY

The valve rangeability required for each application depends on the maximum and minimum Cv as determined from the valve sizing line to the right of the surge line on the compressor characteristic chart (figures 2A and 2B). High performance control valves offer rangeabilities varying from 45:1 to 100:1. These are adequate for the majority of installations but higher rangeabilities can be achieved for exceptional applications.

FIGURE 3. TRACES INDICATING THE TRANSIENT DROP IN FLOW PRECEDING THE PRESSURE SURGE CYCLES



SPEED OF RESPONSE TO SUDDEN PROCESS DISTURBANCES

Studies of gas and vapour surges in turbo-compressor systems reveal that the actual amplitudes of the pressure pulsations are less than the values obtained from calculations using the measured amplitudes of the flow pulsations. A simple relationship between the two is therefore non-existent.

As will be seen from figure 3, which is a synthesis of results from a number of anti-surge systems, the precursor to a surge is an extremely rapid fall in flow followed by a much greater fall at the actual surge. From a control point of view, the difficulties arise with the speed at which these flow changes and related pressure changes take place. The time elapse from the first indication of a surge to the first major flow reversal can be less than 0.07 seconds (see Figure 3). The instrumentation must be able to detect these fast process changes but, equally, the control valve must be capable of following these instructions. This calls for stroking times (closed to open) in the region of 0.5 to 3.0 seconds depending on the length of the valve stroke. Speeds of this order can be achieved without overshoot using, in the case of pneumatic actuators, high capacity volume boosters (relays that increase the rate at which a valve positioner can apply air to the actuator). In most cases only fast opening is required; the closing stroke may take place at a slower speed.

A fast stroking speed does not of itself ensure the valve's instant response to transient disturbances affecting a control system. The ability of a valve to meet this requirement can be tested in a number of ways, the two most important being:

- The application of step change signals
- The application of sinusoidal signals at varying frequencies, the results being plotted on a Bode diagram.

These two procedures are described briefly in the next two sections. For a complete explanation, refer to the appendix.

STEP INPUTS

This form of performance testing involves, as its name implies, the application of step inputs to the control valve/actuator assembly and the measurement of the valve's response. Resolution, dead band, dead time and step response time are the important performance factors that can be gleaned from this test.

In dynamic testing, a control valve with its actuator and positioner behaves in a similar manner to a single capacity system and can therefore be represented by a first order differential equation. In applying such an equation to investigate the dynamic performance of a control valve the units of the input signal must be compatible with the units of the valve response, Q_0 and q_0 . In this case the response is 'valve travel'. The input must therefore be converted from its 'instrument units' to the equivalent valve travel units of length. This can be accomplished by a conversion equation.

Symbols

I	= maximum input signal	instrument units
i_2	= input signal at time $t = 0$	instrument units
i	= input signal at time t	instrument units
Q_{st}	= maximum input signal (I converted)	valve units (mm)
Q_0	= maximum valve response (max. travel)	valve units (mm)
q_{oo}	= valve response (travel) at time $t = 0$	valve units (mm)
q_0	= valve response (travel) at time t	valve units (mm)
q_i	= input signal at time t (i converted)	valve units (mm)
t_s	= time at which steady state conditions have been achieved	seconds
T_c	= time constant	seconds

At 'steady state' conditions ($t = t_s$), I is equivalent to Q_{st} . At $t = 0$, i_2 is equivalent to q_{oo} . For intermediate times (between $t = 0$ and $t = t_s$), the value of the valve units q_i equivalent to i may be calculated from:

$$q_i = \left[\frac{(Q_0 - q_{oo})}{(I - i_2)} \times (i - i_2) \right] + q_{oo}$$

Representing the valve actuator and positioner unit by a first order differential equation:

$$T_c \frac{d(q_0 - q_{oo})}{dt} + (q_0 - q_{oo}) = Q_{st} \quad (1)$$

the solution to this equation is:

$$(q_0 - q_{oo}) = (Q_{st} - q_{oo}) + Ae^{-\frac{1}{T_c}t} \quad (2)$$

(for the complete solution of (2), see appendix)

when $t = 0$, $q_0 = q_{oo}$, therefore $A = - (Q_{st} - q_{oo})$

hence at any time t the fractional change made (FCM) is:

$$FCM = \frac{q_0 - q_{oo}}{Q_{st} - q_{oo}} = 1 - e^{-\frac{1}{T_c}t} \quad (3)$$

when t equals the time constant T_c

$$FCM = \frac{q_0 - q_{oo}}{Q_{st} - q_{oo}} = \left(1 - \frac{1}{2.718} \right) = 0.632 \quad (4)$$

This indicates that the time constant for the valve assembly can be found from the trace of the valve response to the step input. The time constant T_c is defined as the time required for the response to reach 63.2% of its final value.

When t is large compared with T_c , the response q_o will have reached its maximum value Q_o .

Equation (3) then becomes

$$\frac{Q_o - q_{oo}}{Q_{st} - q_{oo}} = 1$$

and so, at steady state conditions, $Q_o = Q_{st}$

In reality, the valve travel at steady state conditions may fall slightly short of the valve's rated travel due to frictional effects which create dead band and hysteresis. Similarly, if Q_{st} is based on the valve's rated travel, then Q_o may fall slightly short of Q_{st} . The time constant however, by definition is 63.2% of the response's achievable final value, which is the slightly reduced Q_{st} . This slight discrepancy is usually ignored as it has practically no meaningful effect on the process performance calculations requiring the step response data.

If, in setting up the test, it is found that the achievable maximum response to the maximum step input is slightly less than the valve rated travel, and this achievable travel is then used for Q_o and also for the calibration of the input signal rather than the rated travel, Q_o should then equal Q_{st} under steady state conditions.

TIME CONSTANT

The time constant T_c is required in calculations of the dynamic performance of process control loops involving control valves. Typical values of the time constant for anti-surge system valves are from 0.32 to 2.25 seconds. It is a direct indication of the speed at which a control valve can react to rapid changes in control signals.

STEP RESPONSE TIME

A further detail concerning the dynamic performance of a control valve is the 'step response time'. This is defined as the interval of time between the initiation of an input step change and the moment the output reaches 86.5% of its final steady state value. This measure of performance therefore takes account of the dead time before response.

In some procedures for calculating the dynamic characteristics of process systems the step response time of the valve plays a major role. The step response time can be found from the plot of the step response test.

The procedure for step input testing and the possible measurements are described in much greater detail in the ISA technical reports in reference 6.



BODE PLOTS (SINUSOIDAL INPUT)

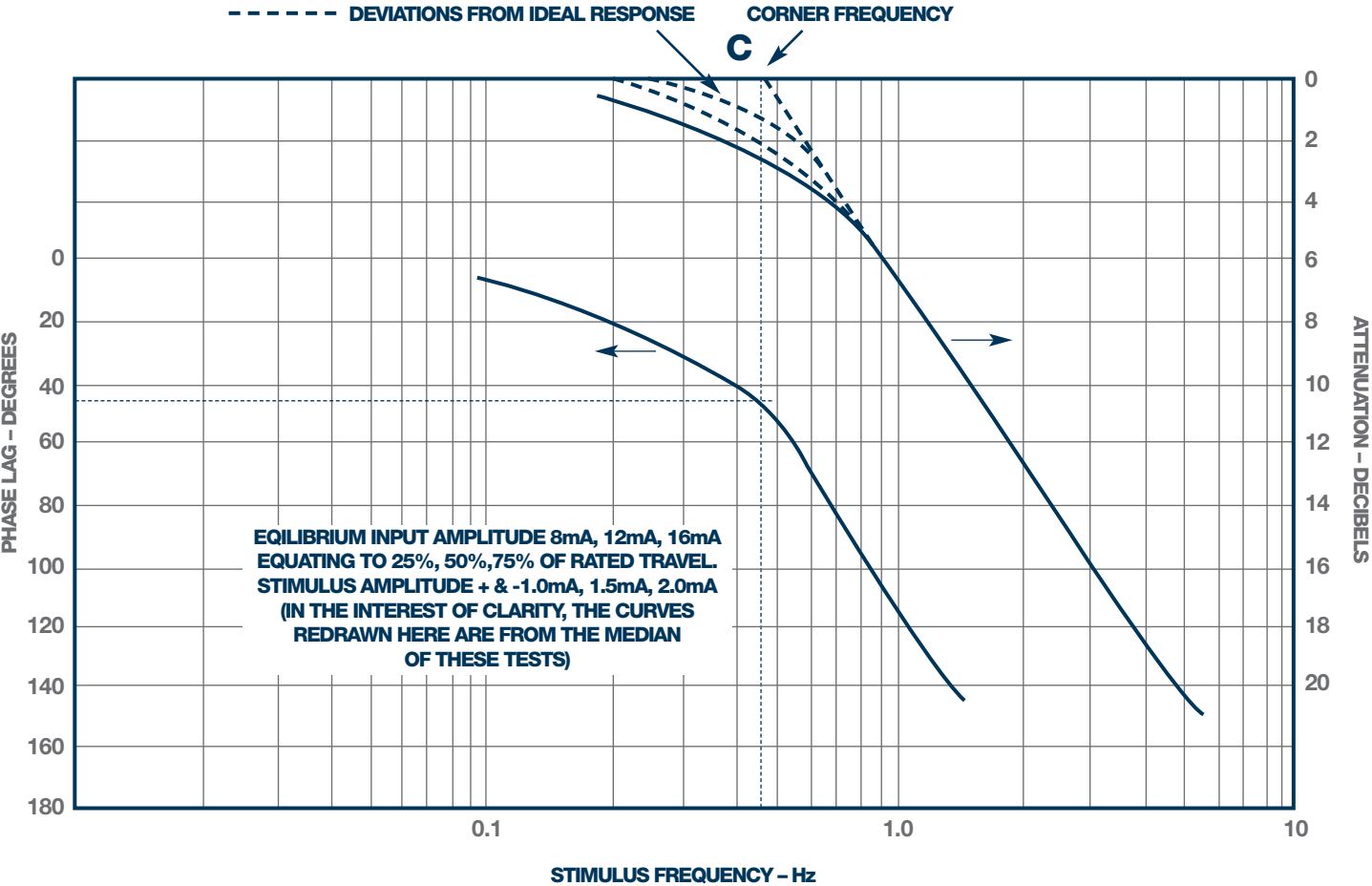
The Bode plot gives some additional information concerning the dynamic performance of a valve, not obtainable from the step input test. The Bode plot represents the response of the valve to the input of a small amplitude sinusoidal signal of specified frequency. This is applied at two or three points within the rated travel of the valve (possibly 25%, 50%, 75%), the amplitude being plus or minus 5% for valves with rated travels not exceeding 70 mm. For valves with higher rated travels the amplitude may be less than 5%. Separate tests are carried out at different points within the valve rated travel and at different frequencies of the input signal. The amplitude ratio for each of these tests is measured and recorded.

The attenuation is calculated in decibels. $dB = 20\log_{10} \frac{Q_o}{Q_i}$

Other measurements which can be made are the wavelengths and the phase lag of the response wave. The attenuation in decibels is plotted on the natural scale against frequency, which is plotted on the log axis of a semi-log chart. On the same chart the phase lag is also plotted on the natural scale against frequency (see figure 4).

In the following example of a first order differential equation representing the valve's response to sinusoidal inputs, the elapsed time 't' commences at the beginning of a cycle under steady state conditions. The input and response are expressed as percentages of their maximum values (i.e. % of Q_i and % of Q_o).

FIGURE 4. BODE DIAGRAM. SOME TYPICAL RESULTS FROM TESTS ON A 300 MM VALVE OPERATED BY A PISTON ACTUATOR WITH AN ELECTRO-PNEUMATIC POSITIONER



SINUSOIDAL INPUT

SYMBOLS

Q_i = maximum value of sinusoidal input (100%)
 q_i = value of input at time t , $q_i = Q_i \sin \omega t$ (% of Q_i)
 f = frequency of input = $1/T$ (s^{-1}) = $\omega/2\pi$
 = frequency of response under steady state conditions
 f_c = corner frequency (s^{-1})
 T_c = time constant for valve assembly $T_c = 1/2\pi f_c$ (s)
 T = periodic time of input signal and of response under steady state conditions (s)
 $T = 2\pi/\omega$ (s)
 t = elapsed time from beginning of each cycle under steady state conditions (s)
 α = angle of lag of response under steady state conditions (radians or degrees)
 $\alpha = \tan^{-1} \omega T_c$ (s)
 Q_o = maximum response (100%) when $(\omega t - \alpha) = \pi/2$
 q_o = response at time t (% of Q_o)
 at time $t = 0$ (steady state conditions established)

$$q_i = 0 \text{ and } q_o = - \left(100 \times \frac{\omega T_c}{\sqrt{1 + (\omega T_c)^2}} \right) \% \text{ of } Q_o$$

$$\text{at time } t = \frac{\alpha}{\omega} = \frac{\tan^{-1} \omega T_c}{\omega}, q_o = 0$$

As explained in the section dealing with step inputs, a control valve with its actuator and positioner reacts in a similar manner to a single capacity process. The first order differential equation representing the dynamic response of a single capacity process to the application of a sinusoidal input is:

$$T_c \frac{dq_o}{dt} + q_o = Q_i \sin \omega t \quad (5)$$

This type of equation can be solved in a number of ways, the principal ones being 'frequency response' and 'direct integration'.

Since the control valve assembly behaves like a single capacity process, the attenuation curve can be reproduced by extending a line from the corner frequency in figure 4, downwards with a slope of -6 dB per octave of frequency. In actuality, as test results will show, the attenuation curve does not connect with the 0 dB attenuation line – the slope starts to decrease at attenuation values between 4 and 5 dB, crossing the corner frequency line at 3 dB and its progress towards the 0 dB line is then asymptotic. These conditions are for a theoretical single stage capacity process. Valves, although they are considered to be single capacity systems, do have present some idiosyncrasies, causing the actual attenuation curve to deviate slightly from the ideal.

In compressor anti-surge applications, it is highly desirable for the phase lag curve to be located towards the right on the Bode diagram where it registers higher frequencies. This gives a higher corner frequency and a lower time constant with faster valve action and a short phase lag.

Full details of the solutions to the differential equation representing the response of a single capacity system to a sinusoidal input, using the frequency response and the integration methods, can be found in the appendix.

SOLUTION BY THE FREQUENCY RESPONSE METHOD, AVOIDING INTEGRATION

Rewriting equation (5) in its transformed version for a first order differential equation where d/dt is replaced by the complex variable 's', gives the transfer function of the system.

$$\frac{Q_o}{Q_i} = \frac{1}{1 + sT_c} \quad (6)$$

The steady state solution to the equation for a single capacity system with a sinusoidal input is satisfied by substituting $j\omega$ (or $j2\pi f$) for 's' in the transfer function.

$$\left(j = \sqrt{-1} \right)$$

$$\frac{Q_o}{Q_i} = \frac{1}{1 + j2\pi f T_c} \quad (7)$$

The modulus $| |$ of this complex number gives the amplitude ratio Q_o/Q_i and the argument \angle gives the phase lag α .

$$\left| \frac{Q_o}{Q_i} \right| = \frac{1}{\sqrt{1 + (2\pi f T_c)^2}} \quad \angle \alpha = \tan^{-1}(-2\pi f T_c) \quad (8)$$

and, by definition the gain or attenuation is $20 \log \frac{Q_o}{Q_i}$ decibels

$$\text{attenuation} = -20 \log \sqrt{1 + (2\pi f T_c)^2} \text{ decibels} \quad (9)$$

$$\text{phase lag } \alpha = \tan^{-1} 2\pi f T_c \text{ radians} \quad (13)$$

when $\alpha = \pi/4$, $f = f_c$, $1 = 2\pi f_c T_c$ and $T_c = \frac{1}{2\pi f_c}$ where f_c is the corner frequency (see Figure 4).

NOISE AND VIBRATION

Although compressors have a high potential for noise generation, the manufacturers are required to comply with noise limitations set by the plant designers and health and safety authorities.

By the same token the anti-surge valve must comply with similar noise limitations and must have the ability to control high pressure gas systems with the minimum of noise and vibration. Some types of vibration can be the result of mechanical problems, but generally on these systems vibration is attributable to aerodynamically generated noise, caused by uncontrolled high fluid velocity which, if uncontrolled, may set up serious vibrations in the adjacent pipework. The compressor anti-surge service requires the control valve to reduce high pressures to very low pressures, the P_1/P_2 ratio being in excess of two, noting that pressure ratios of 7.5 are sometimes needed. Sonic velocity is produced by a P_1/P_2 ratio of approximately 2:1 and this is usually noisy: higher ratios produce supersonic velocities with resultant shock waves and these are very noisy. Conventional single stage control valves on compressor recycle applications have been known to emit noise levels in excess of 122 dBA measured at 1 metre downstream and 1 metre from the pipe. For a single stage pressure reduction (compressible fluid), the trim exit velocity is proportional to $\sqrt{2 \times \text{enthalpy drop}} = \sqrt{2 \times (H_1 - H_{2T})}$ (refer to figure 5A).

From this enthalpy/entropy chart it can be seen that if the pressure drop is divided into a number of stages, each with an enthalpy drop of approximately $H_1 - H_2$, the jet velocity at the trim exit and any part within the trim will be reduced and, with a sufficient number of stages, the velocity levels can be restricted to something well below sonic, restricting noise levels to 85 dBA or less.

SOME PRACTICAL SOLUTIONS

A valve capable of this type of performance is the multistage valve which incorporates the concentric multi-sleeve trim, as shown in figure 6. The trim consists of a number of sleeves which are drilled with small holes of varying size.

There are annular gaps between each sleeve and, within these gaps, spacers divide the holes into separate rows. The gaps allow partial pressure recovery between each discrete stage and the holes do not coincide from one sleeve to another. Thus, the fluid is constrained to flow through the holes and, in the annular gaps, it must change direction twice before entering the holes in the next sleeve. This means that a trim with four discrete stages has a further six pressure reduction stages through the turns between each stage. Ideally, the trim should be specially designed for each service but, in general, it is desirable to restrict the jet velocity at the first stage to no more than a Mach number of 0.70 and to reduce this through each successive stage until the last one, where the jet velocity is preferably no higher than 0.45 Mach. For effective noise reduction the holes should not exceed 4 mm in diameter, with 3 mm the preferred size. Experience suggests that there can be flexibility in the arrangement and size of the holes; a device that can be useful in giving the valve a purpose designed flow/lift characteristic. An additional feature of this design is its ability to handle fluids containing particles of foreign matter without blockage. Experience suggests that to avoid vibration problems in the downstream piping the fluid velocity in the valve outlet section should not exceed Mach 0.3.

Low noise trims must also include design features to avoid mechanical vibration which could result from the impact of high velocity jets and turbulent fluid. Mechanical clearances in plug and stem guides must be carefully determined to avoid lateral vibration whilst recognising the effects of temperature changes. Elimination of any axial vibration of the plug and stem assembly is provided by the pressure balanced plug design.

Besides equalising pressures above and below the plug the design incorporates a balancing seal which imparts a damping action to discourage any tendencies towards axial vibration. Powerful actuators also assist towards axial stability of the valve plug.

FIGURE 5A. ENTHALPY/ENTROPY CHART, ILLUSTRATING THERMODYNAMIC CHANGES THAT OCCUR WITH COMPRESSIBLE FLOW THROUGH A SINGLE STAGE VALVE AND A MULTISTAGE VALVE WITH THREE STAGES AND FOUR PATH TURNS

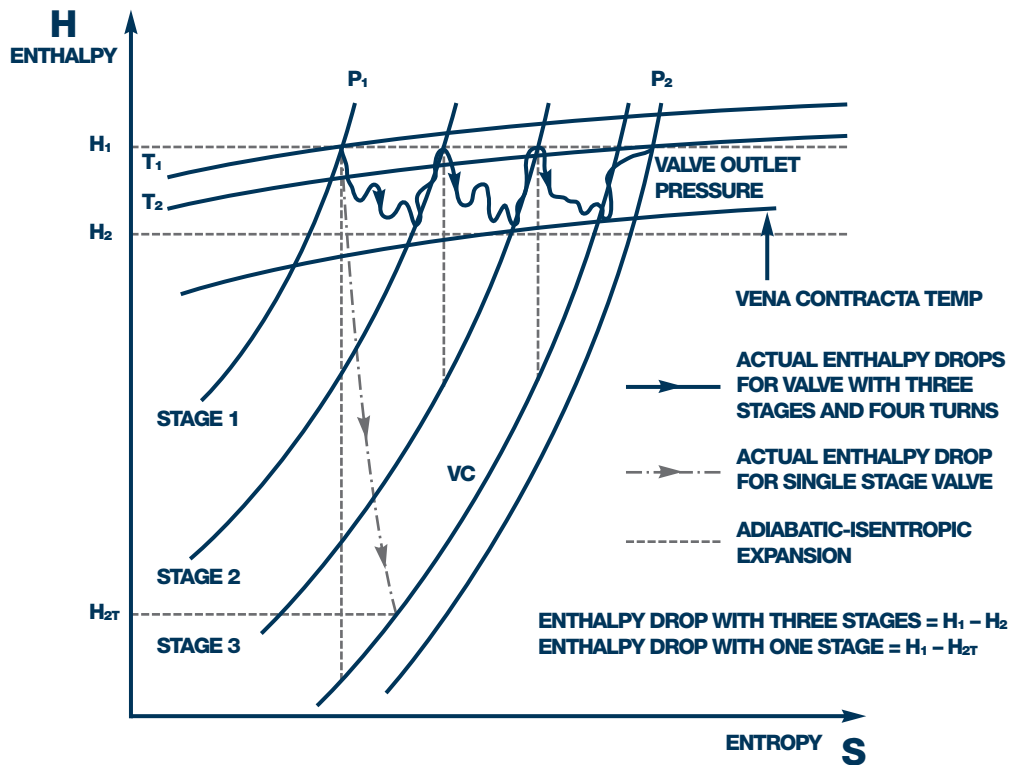


FIGURE 5B. ENTHALPY/ENTROPY CHART, ILLUSTRATING COMPRESSIBLE FLOW THROUGH A LABYRINTH TYPE CONTROL VALVE. THE ACTUAL POLYTROPIC EXPANSION (1) IS PRODUCED WITH A LARGE NUMBER OF TURNS IN THE FLOW PATH AND (2) WITH A SMALLER NUMBER

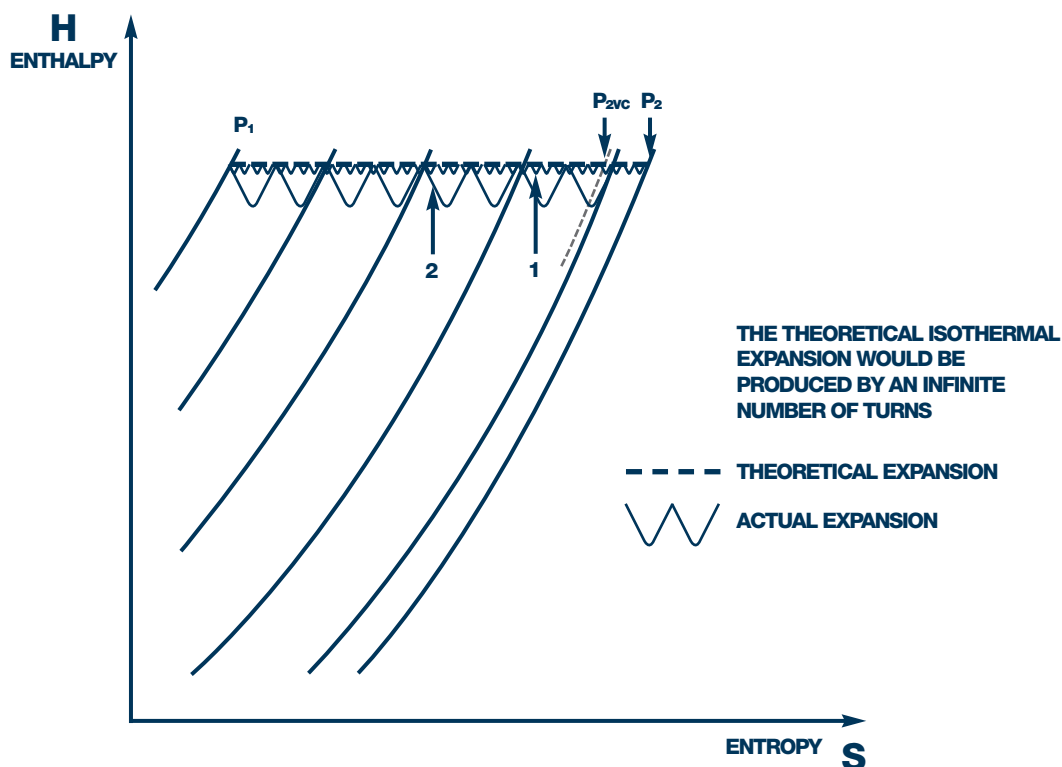
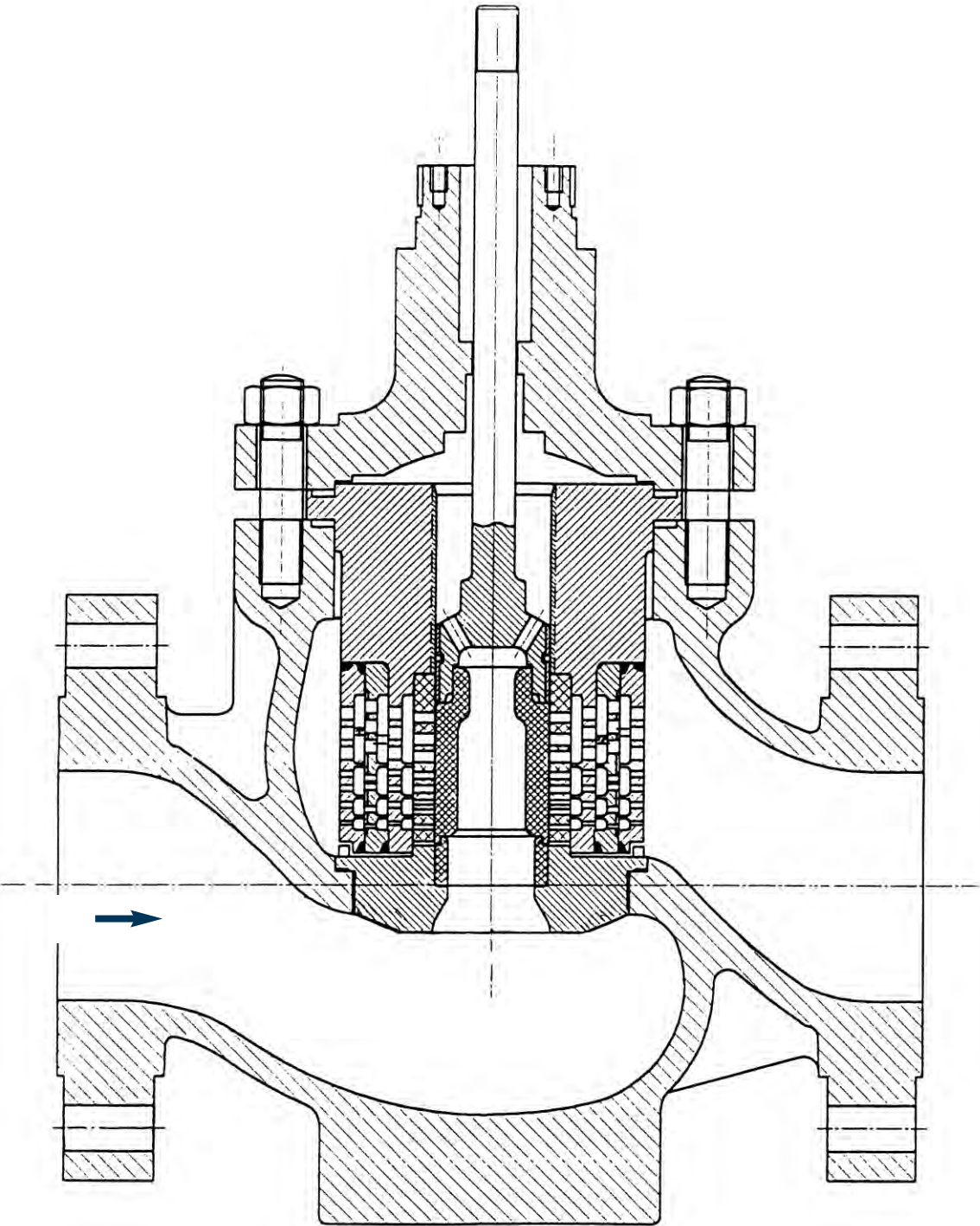


FIGURE 6. MULTI-STAGE CONTROL VALVE WITH FOUR DISCRETE STAGES AND SIX TURNS OF FLOW PATH



(Courtesy of Koso kent Introl Ltd)

Another type of multistage trim is illustrated in figure 7. This is generally known as the labyrinth trim because it consists of a number of discs stacked on top of each other, each disc having etched or milled into one side a large number of small flow passages. The passages have a varying number of right angle turns and the flow area increases from the inlet to the outlet.

The working principle is similar to the concentric sleeve design but there are no significant gaps between stages and there is therefore no significant pressure recovery. The design, because of height of the stacked discs to give the required Cv, demands a longer rated travel than the concentric sleeve design, so greater attention has to be made to the provision of fast opening. As will be seen from figure 5b, the greater number of turns available in this design reduces the enthalpy drop most effectively, so very low jet velocities can be achieved. The number of turns is determined with the intention of restricting the fluid velocity within the trim to something between 0.5 and 0.3 Mach. To accommodate a stacked disc design with a large number of turns, a larger valve body may be required and, with a large number of discs, the actuator will require a longer stroke. The anti-vibration design features of this valve are very similar to those of the concentric sleeve design. Both the concentric sleeve and the labyrinth low noise valves have a proven record on compressor anti-surge systems.

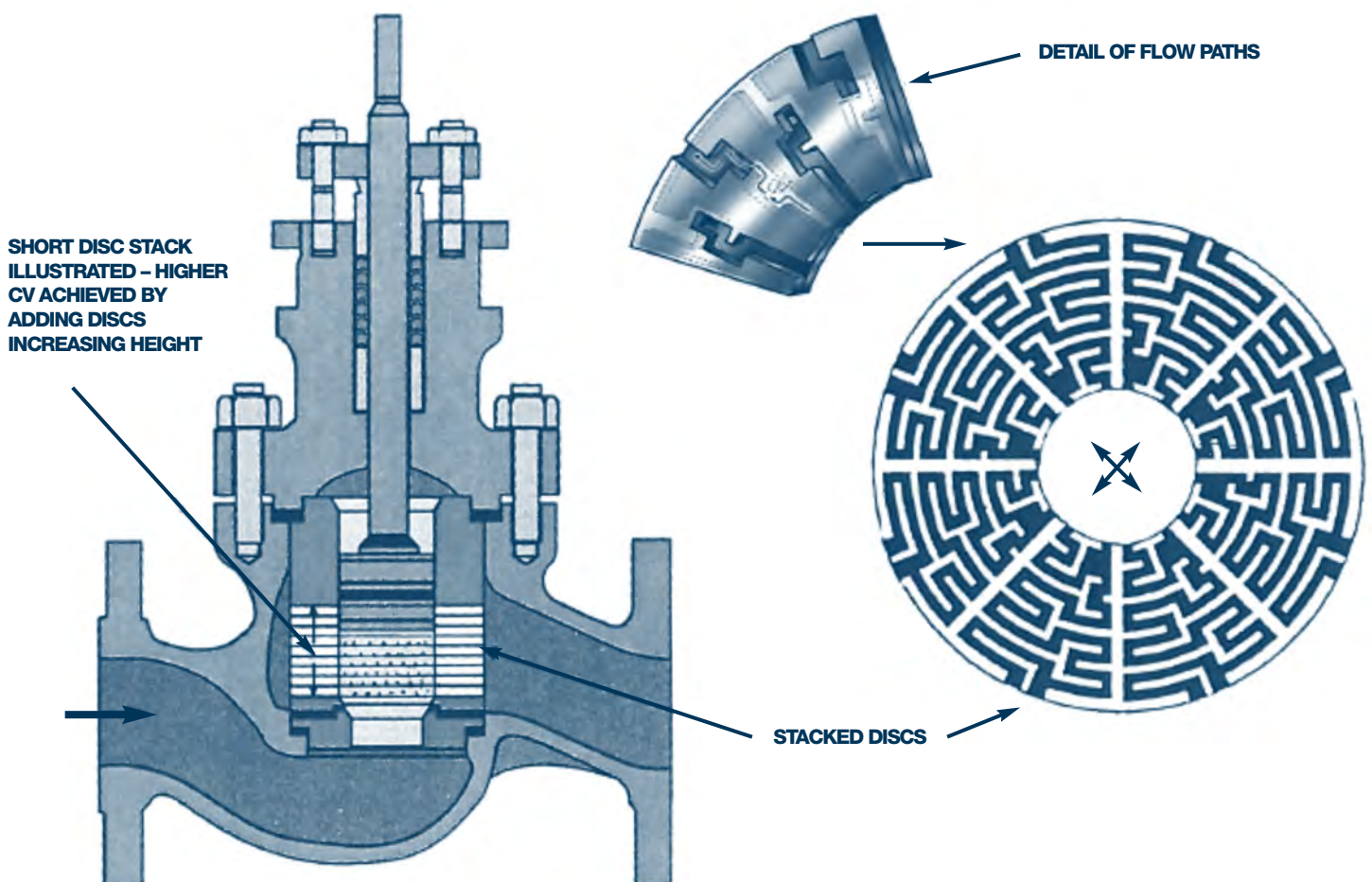
LEAKAGE IN CLOSED POSITION

In some processes, the compressor anti-surge valve, under normal plant operation, is set up to come into operation at 20 to 30% of the maximum flow. This ensures stable compressor operation but it does not necessarily avert surges and it is costly on energy. A recycling valve used in this way does not need to have tight shut-off. IEC 543-4 Class II (ANSI/FCI 70-2 Class II) is adequate.

The preferred method of operating a recycling valve is to maintain the valve in a closed position until the instrumentation detects that the compressor is operating quite close to the surge line (figure 2A). The valve is then commanded to open quickly. During the periods of stable operation, when the valve is required to remain closed, very tight shut-off is required.

The maximum limits set by IEC 534-4 Class IV and Class V (ANSI/FCI 70-2 Class IV and Class V) can be achieved with metal-to-metal seating and, depending on service conditions, a resilient seat design is available to achieve a shut-off leakage better than Class V.

FIGURE 7. TORTUOUS PATH (LABYRINTH) TYPE VALVE – THERE ARE A NUMBER OF VARIATIONS BASED ON THIS PRINCIPLE



ACTUATOR

In compressor recycle services for surge protection, fast stroke speeds are important.

Actuators can be designed and fitted with relays that will achieve the required speeds. The concentric sleeve design does not have a high rated travel so the reduction in time to stroke the valve from the closed to the fully open position is not such a difficult task as with some low noise valves that have a longer valve travel. Opening quickly is essential. As stated in the *speed of response* section, opening times can be very short. For example 0.5 to 2.0 s can be achieved for short travel valves and 1.5 to 3.5 s for longer travels.

Actuators are invariably of the pneumatic power cylinder type, working with air pressures in the range from 350 kPa.g to 750 kPa.g.

Normally, anti-surge valves are required to open in the event of failure of the actuator operating supply but, occasionally, in those systems where the valve is constantly recycling a portion of the process fluid, the valve may be required to lock in the position in which it was operating at the instant of the failure.

EXAMPLES OF CONTROL VALVES SPECIFIED FOR COMPRESSOR ANTI-SURGE SERVICES

Unless otherwise stated pressures are 'absolute'.

APPLICATION 1. CONCENTRIC SLEEVE VALVE

Service conditions at 12 points along the agreed 'valve line' were the basis for Cv and noise calculations to ensure the valve provides adequate protection throughout the full range of compressor speeds and gas flows.

This typical example is for the maximum flow condition:

SERVICE CONDITIONS

Fluid	Hydrocarbon Gas
Flow	173,000 kg/hr
Molecular Weight	19.70 kg/kg mol
Specific Heat Ratio	1.25
Flowing Temperature	30 C
Inlet Pressure	2.55×10^3 kPa
Downstream Pressure	1.38×10^3 / 0.93×10^3 kPa
Upstream Piping	500 mm sch 60 wall thickness 20.60 mm
Downstream Piping	500 mm sch 60 wall thickness 20.60 mm
Leakage in closed position to be stated	not to exceed ANSI/FCI 70-2 Class V
Action on air failure	valve opens
Time for full opening stroke	not to exceed 2.6 s
Noise at 1 m must not exceed	85 dBA

A valve with the following specification qualifies for these service conditions.

APPLICATION 1. (CONT'D)

VALVE SPECIFICATION

Globe style valve	
Valve inlet	300 mm
Valve outlet	300mm Body rating ANSI Class 600
Body material	Cast Inconel ASTM A494 CW-6MC76
Inlet connection	Flanged ANSI 600
Outlet connection	Flanged ANSI 600
Trim	Concentric sleeve design with three discrete stages (seven pressure reduction stages) seal balanced plug with heavy section PTFE seal
Rated Cv of valve	450 (= 389.24 Kv)
Rangeability	85:1
Trim material	Inconel 625 with stellite facing
Leakage flow in closed position	within IEC 534-4 Class V (ANSI/FCI 70-2 Class V)
Power conversion at maximum flow	2.44 Mw
Noise prediction at max flow	82.5 dBA at 1 m downstream and 1 m from pipe
Actuator	Pneumatic power cylinder, size 200, double acting
Time for full opening stroke	1.8 secs
Action on air failure	valve opens

This valve is similar to a North Sea platform application where NACE material compliance is required. The sour gas services involved in the North Sea operations are an example of difficult applications requiring a sound knowledge of compatible materials.



APPLICATION 2. LABYRINTH VALVE

SERVICE CONDITIONS

Four sets of operating conditions were given – two are quoted here.

CASE 1.

Compressor speed	122%
Flow	156.468 SCMh
Temp	33 C
MW	24.6
P ₁	1,210 kPa
P ₂	293 kPa
Calculated Cv	895 US units (=774 Kv)
Max allowable opening time	2.5 s

CASE 3.

Compressor speed	50%
Flow	95.085 SCMh
Temp	33 C
MW	24.6
P ₁	493 kPa
P ₂	348 kPa
Calculated Cv	2,085 US units (= 1,804 Kv)

VALVE SPECIFICATION

Valve body style	Angle valve
Valve size	455 x 610 mm
Trim	Stacked discs – with labyrinth flow paths – 22 turns
Rated Cv	2,180 US units (= 1,886 kv)
Valve travel	510 mm
Time for opening	2.0 s
Power conversion at maximum flow	1.65 Mw
Noise prediction at max flow	81.0 dBA at 1 m downstream and 1m from pipe
Actuator	pneumatic double acting cylinder with electro/pneumatic positioner and volume boosters
Air failure action	valve opens

CONCLUSIONS

The control valve is, compared with the instrumentation, a slow developer.

New design features have been incorporated over the years but not at the rate or the degree of innovation affecting the instruments that measure the process changes and command the valve to take the corrective action. However, for the applications engineer, the control valve presents in some ways greater difficulties than the instrumentation when attempting to prescribe the ideal specification for severe services such as compressor anti-surge bypass. The valve is at the 'business end' of the control system and has therefore, next to the actual compressor, the most responsible role. It has the direct and final effect on the fluid being controlled. It is unusual for the same working conditions to be repeated from one plant to another, so the valves are to a great extent tailor-made for each application. Such valves are expensive single units of plant and it is therefore imperative that the application engineering is of the highest order since mistakes can only come to light after the plant start-up and subsequent corrective action can be very expensive for all parties.

Some of the considerations necessary, before a control valve can be specified for compressor bypass service, have been outlined in this paper which, for the most part, has been written at a basic level for those not too familiar with control valve technology.

Thanks are extended to Dr J. T. Turner for correcting the proofs and making some valuable suggestions.



APPENDIX

The appendix includes the fundamental definitions of the valve sizing coefficient C_v (K_v), the flow/lift characteristic and the pressure recovery factor F_L . For the benefit of those who are no longer familiar with the solution of differential equations and don't have their maths books to hand, but are curious to know the methods used to arrive at the results, these are reproduced in the appendix.

C_v and K_v

These are the valve flow coefficients used to simplify the sizing equations (see references 8 and 9). The only difference between the two is in the units chosen for the service conditions.

The definition for C_v is the flow in US gallons per minute of water at 60 F, with a pressure drop across the valve of 1 psi (6,895 pa). The units of C_v are therefore US gallons/minute.

The definition for K_v is the flow in cubic metres of water at 10.6 C with a pressure drop across the valve of 1 bar (1×10^5 pa). The units of K_v are therefore cubic metres/hour.

It will be seen from references 8 and 9 that these flow coefficients, although based on liquid flow, are used in sizing equations for gases and vapours, made possible by the introduction of an expansion factor. The procedures for testing valves to allocate C_v and K_v values can be found in reference 10.

F_L

This is termed the pressure recovery factor. It allows the pressure recovery, which usually takes place between the vena contract and the valve outlet, to be applied when sizing valves operating above the critical pressure drop. For normal flow conditions the pressure recovery is included in the valve C_v and K_v (see references 9 and 10).

Flow/lift characteristic

Assuming there is a constant pressure drop across it, a control valve has a predetermined relationship between the fluid flowing through it and the amount of the valve opening, usually termed the 'lift'. This relationship, or characteristic, depends on the design of the valve plug and seat, known collectively to valve engineers as the 'trim'. The most important feature is the way in which the orifice area varies with the valve lift. The two most frequently used characteristics in process control are 'linear' and 'equal percentage'.

As its name implies, the linear characteristic gives a linear relationship between flow and lift, whilst the equal percentage gives an exponential relationship such that at any point in the valve lift, a further incremental increase in the lift gives an equal percentage increase in flow (see reference 11).



SOLUTION OF THE DIFFERENTIAL EQUATIONS

STEP INPUT CHANGE

Solution of the differential equation. (The key to the symbols is in the main text.)

$$T_c \frac{d(q_o - q_{oo})}{dt} + (q_o - q_{oo}) = Q_{st}$$

$$\frac{d(q_o - q_{oo})}{dt} + \frac{(q_o - q_{oo})}{T_c} = \frac{Q_{st}}{T_c}$$

multiply each side by the integrating factor $e^{\int \frac{1}{T_c} dt} = e^{\frac{1}{T_c} t}$

$$e^{\frac{1}{T_c} t} \frac{d(q_o - q_{oo})}{dt} + \frac{1}{T_c} e^{\frac{1}{T_c} t} (q_o - q_{oo}) = \frac{Q_{st}}{T_c} e^{\frac{1}{T_c} t}$$

this can be rewritten as:

$$\frac{d}{dt} \left[e^{\frac{1}{T_c} t} (q_o - q_{oo}) \right] = \frac{1}{T_c} Q_{st} e^{\frac{1}{T_c} t}$$

Integrating

$$e^{\frac{1}{T_c} t} (q_o - q_{oo}) = \frac{1}{T_c} \int (Q_{st} - q_{oo}) e^{\frac{1}{T_c} t} dt + A \quad \text{'A' is a constant of integration}$$

$$(q_o - q_{oo}) = \frac{1}{T_c} e^{-\frac{1}{T_c} t} \int (Q_{st} - q_{oo}) e^{\frac{1}{T_c} t} dt + A e^{-\frac{1}{T_c} t}$$

$$(q_o - q_{oo}) = \frac{1}{T_c} e^{-\frac{1}{T_c} t} (Q_{st} - q_{oo}) T_c e^{\frac{1}{T_c} t} + A e^{-\frac{1}{T_c} t}$$

$$(q_o - q_{oo}) = (Q_{st} - q_{oo}) + A e^{-\frac{1}{T_c} t} \quad \text{when } t = 0 \quad q_o = q_{oo} \quad 0 = (Q_{st} - q_{oo}) + A$$

$$A = -(Q_{st} - q_{oo})$$

$$(q_o - q_{oo}) = (Q_{st} - q_{oo}) - (Q_{st} - q_{oo}) e^{-\frac{1}{T_c} t}$$

$$FCM = \frac{(q_o - q_{oo})}{(Q_{st} - q_{oo})} = 1 - e^{-\frac{1}{T_c} t} = \text{fractional change made}$$

When $t = T_c$ $FCM = 0.632$ For a single capacity process the time constant is the time required for the response to reach 63.2% of its final value.

SOLUTION TO THE EQUATION BY THE FREQUENCY RESPONSE METHOD

SINUSOIDAL INPUT SIGNAL

Symbols as shown in the text.

$$T_c \frac{dq_o}{dt} + q_o = Q_i \sin \omega t \quad (5)$$

Rewriting equation (5) in its transformed version for a first order differential equation where d/bt is replaced by the complex variable 's' gives the transfer function of the system.

$$\frac{Q_o}{Q_i} = \frac{1}{1 + sT_c} \quad (6)$$

The steady state solution to the equation for a single capacity system with a sinusoidal input is satisfied by substituting $j\omega$ (or $j2\pi f$) for 's' in the transfer function.

$$\left(j = \sqrt{-1} \right)$$

$$\frac{Q_o}{Q_i} = \frac{1}{1 + j2\pi fT_c} \quad (7)$$

To arrive at the required solution this complex number must be divided into its real and imaginary parts.

$$\left| \frac{Q_o}{Q_i} \right| = \frac{1}{1 + j2\pi fT_c} \times \frac{1 - j2\pi fT_c}{1 - j2\pi fT_c} = \frac{1 - j2\pi fT_c}{1 + (2\pi fT_c)^2} = \frac{1}{1 + (2\pi fT_c)^2} - j \frac{2\pi fT_c}{1 + (2\pi fT_c)^2}$$

The modulus $| |$ of this complex number gives the amplitude ratio Q_o/Q_i and the argument gives \angle the phase lag α .

$$\left| \frac{Q_o}{Q_i} \right| = \frac{1}{\sqrt{1 + (2\pi fT_c)^2}} \quad (8) \quad \angle \alpha = \tan^{-1}(-2\pi fT_c)$$

and, by definition the gain or attenuation is $20 \log \frac{Q_o}{Q_i}$ decibels

$$\text{attenuation} = -20 \log \sqrt{1 + (2\pi fT_c)^2} \text{ decibels} \quad (9)$$

$$\text{phase lag } \alpha = \tan^{-1} 2\pi fT_c \text{ radians} \quad (13)$$

when $\alpha = \pi/4$, $f = f_c$, $1 = 2\pi f_c T_c$ and $T_c = \frac{1}{2\pi f_c}$ where f_c is the corner frequency (see Figure 4).

SOLUTION OF THE DIFFERENTIAL EQUATION BY INTEGRATION

SINUSOIDAL INPUT SIGNAL

The key to the symbols is in the main text.

$$T_c \frac{dq_o}{dt} + q_o = Q_i \sin \omega t$$

$$\frac{dq_o}{dt} + \frac{q_o}{T_c} = \frac{Q_i}{T_c} \sin \omega t$$

multiplying throughout by the integrating factor $e^{\int \frac{1}{T_c} dt} = e^{\frac{1}{T_c} t}$

$$e^{\frac{1}{T_c} t} \frac{dq_o}{dt} + \frac{1}{T_c} e^{\frac{1}{T_c} t} q_o = \frac{Q_i}{T_c} e^{\frac{1}{T_c} t} \sin \omega t$$

this can be written as:

$$\frac{d}{dt} \left[e^{\frac{1}{T_c} t} q_o \right] = \frac{Q_i}{T_c} e^{\frac{1}{T_c} t} \sin \omega t$$

$$e^{\frac{1}{T_c} t} q_o = \frac{1}{T_c} \int Q_i e^{\frac{1}{T_c} t} \sin \omega t dt + A$$

rewriting

$$q_o = \frac{Q_i}{T_c} e^{-\frac{1}{T_c} t} \int e^{\frac{1}{T_c} t} \sin \omega t dt + A e^{-\frac{1}{T_c} t} \quad (a)$$

For steady state conditions t is large compared with T_c and the last term then diminishes to almost zero and may be ignored. Under these conditions the response (the valve) oscillates at the same frequency as the input but not in phase. It will lag behind the input at an angle $\alpha = \tan^{-1} \omega T_c = \tan^{-1} 2\pi f T_c$. At constant frequency the phase lag is constant.

The integral can be solved by using integration by parts $\int u \cdot dv = uv + \int v du$
rewriting to facilitate integration by parts

$$q_o = \frac{Q_i}{T_c} e^{-\frac{1}{T_c} t} \left[\int e^{\frac{1}{T_c} t} (\sin \omega t) dt \right]$$

making $u = e^{\frac{1}{T_c} t}$; $dv = \sin \omega t$; $v = -\frac{1}{\omega} \cos \omega t$ and integrating the bracket

$$-e^{\frac{1}{T_c} t} \frac{1}{\omega} \cos \omega t + \int \frac{1}{\omega} \cos \omega t \frac{1}{T_c} e^{\frac{1}{T_c} t} dt$$

integrating again and making $u = e^{\frac{1}{T_c} t}$; $dv = \frac{1}{\omega} \cos \omega t dt$; $v = \frac{1}{\omega} \sin \omega t$

$$-e^{\frac{1}{T_c} t} \frac{1}{\omega} \cos \omega t + \frac{e^{\frac{1}{T_c} t}}{T_c} \frac{1}{\omega^2} \sin \omega t - \int \frac{e^{\frac{1}{T_c} t}}{T_c^2} \frac{1}{\omega^2} \sin \omega t$$

SOLUTION OF THE DIFFERENTIAL EQUATION BY INTEGRATION (CONT'D)

SINUSOIDAL INPUT SIGNAL

rewriting

$$\int e^{\frac{1}{T_c} t} \sin \omega t \, dt = e^{\frac{1}{T_c} t} \frac{1}{\omega} \left[\frac{1}{\omega T_c} \sin \omega t - \cos \omega t \right] - \frac{1}{\omega T_c^2} \int e^{\frac{1}{T_c} t} \sin \omega t \, dt$$

rearranging

$$\int e^{\frac{1}{T_c} t} \sin \omega t \, dt = \frac{T_c e^{\frac{1}{T_c} t}}{\omega^2 T_c^2} \frac{(\sin \omega t - \omega T_c \cos \omega t)}{\left(\frac{1}{\omega^2 T_c^2} + 1 \right)}$$

since $\tan \alpha = \omega T_c = \frac{\sin \alpha}{\cos \alpha}$ and $\cos \alpha = \frac{1}{\sqrt{1 + \omega^2 T_c^2}}$ this can be simplified to:

$$\int e^{\frac{1}{T_c} t} \sin \omega t \, dt = T_c e^{\frac{1}{T_c} t} \frac{(\sin \omega t \cos \alpha - \cos \omega t \sin \alpha)}{\sqrt{1 + \omega^2 T_c^2}} = T_c e^{\frac{1}{T_c} t} \frac{\sin(\omega t - \alpha)}{\sqrt{1 + \omega^2 T_c^2}}$$

this can be substituted in equation (a) for the integral term, giving:

$$q_o = Q_i \frac{\sin(\omega t - \alpha)}{\sqrt{1 + \omega^2 T_c^2}}$$

When $(\omega t - \alpha) = \pi/2$, $q_o = Q_o$, so the amplitude ratio is:

$$\frac{Q_o}{Q_i} = \frac{1}{\sqrt{1 + \omega^2 T_c^2}} = \frac{1}{\sqrt{1 + (2\pi f T_c)^2}}$$

REFERENCES AND BIBLIOGRAPHY

1. R.A.Strub *Anti-surge regulation for compressors, published by Brown Boveri-Sulzer Turbomachinery Ltd.*
2. A.S.Clark *Evaluation of a control valve for anti-surge control systems, published by British Gas Engineering Research.*
3. N.Staroselsky & L. Ladin *Improved surge control for centrifugal compressors, published by Chemical Engineering.*
4. O.Kneisel *The frequency response method as a means of analysis of control problems in process industries, published by Hammel-Dahl Co.*
5. T.Fugita, T.Kagawa & M.Kuroda *Disturbance characteristics of control valve positioners, published by ISA Process Measurement & Control Division 2002.*
6. ANSI/ISA *Test procedure for control valve response measurement from step inputs, ANSI/ISA – 75.25.01-2000.*
Control valve response measurement from step inputs, ANSI/IS - 75.25.02 - 2000, published by ISA, PO Box 12277, Research Triangle Park, NC 27709.
7. E.W. Singleton *The role of the control valve in improved process control, published by ABB Control Valves. Reprinted from Valve World, volume 4, Issue 5, October 1999.*
8. IEC *Control valve terminology, IEC Standard 60534-1, published by Commission Electrotechnique Intenationale, 3 Rue deVarembé, Case postale 131, 1211 Genève 20, Suisse.*
9. IEC *Control valve sizing equations, IEC Standard 60534-2-1.*
10. IEC *Control valve flow capacity test procedures, IEC Standard 60534-2-3.*
11. IEC *Control valve inherent flow characteristics and rangeablty, IEC Standard 60534-2-4.*
12. IEC *Control valve inspection and routine testing, IEC standard 60534-4.*
13. ANSI/FCI *Control valve seat leakage, ANSI/FCI standard 70-2.*
14. E.W. Singleton *The derivation of the IEC/ISA control valve sizing equations, published by Koso Kent Introl Ltd. 1980.*



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